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TOPSIS for bi-level MODM problems

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ABSTRACT

TOPSIS (technique for order preference by similarity to ideal solution) is a multiple criteria method to identify solutions from a finite set of alternatives based upon simultaneous minimization of distance from an ideal point and maximization of distance from a nadir point. This paper proposes a fuzzy TOPSIS algorithm to solve bi-level multi-objective decision-making (BL-MODM) problems, and in which the objective function at each level are non-linear functions which are to be maximized. The proposed model for getting the satisfactory solution of the BL-MODM problems includes the membership functions for the upper level decision variables vector with possible tolerances, the membership function of the distance function from the positive ideal solution (PIS) and the membership function of the distance function from the negative ideal solution (NIS). A numerical illustrative example is given to clarify the proposed TOPSIS approach of this paper.

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1. Introduction

Technique for order performance by similarity to ideal solution (TOPSIS), one of the known classical multiple criteria decision making (MCDM) method, bases upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). It was first developed by Hwang and Yoon [1] for solving a multiple attribute decision making problem. A similar concept has also been pointed out by Zeleny [2]. Lia et al. [3] extended the concept of TOPSIS to develop a methodology for solving multiple objective decision making (MODM) problems. Recently, Abo-Sinna [4] extended TOPSIS approach to solve multi-objective dynamics programming (MODP) problems. As he showed that using the fuzzy max–min operator with non-linear membership functions, the obtained solutions are always non-dominated by the original MODP problems. Further extension of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure was presented by Abo-Sinna et al. in [5,6]. Deng et al. [7] formulated the inter-company comparison process as a multi-criteria analysis model, and presented an effective approach by modifying TOPSIS for solving such a problem. Chen [8] extended the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision-making problems in a fuzzy environment and he defined the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS).

Generally, TOPSIS provides a broader principle of compromise for solving multiple criteria decision making problems. It transfers m-objectives (criteria), which are conflicting and noncommensurable, into two objectives (the shortest distance from the PIS and the longest distance from the NIS). They are commensurable and most of time conflicting. Then, the bi-objective problem can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS's compromise solution by a second-order compromise. The max–min operator is then

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considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the PIS and the longest distance from the NIS) [5,6,3].

A bi-level programming problem (BLPP) is formulated for a problem in which two decision makers (DMs) make decisions successively. The two decision makers are located at two different hierarchical levels, each independently controlling one set of decision variables, with different and perhaps conflicting objectives. In the hierarchical decision process, the lower-level DM (LLDM) executes his/her decision powers, after the decisions of the upper-level DM (ULDM). Although the ULDM independently optimizes its own benefits, the decision may be affected by the reaction of the LLDM. As a consequence, decision deadlock arises frequently and the problem of distribution of proper decision power is encountered in most of the practical decision situations. In other words, the basic concept of the bi-level programming technique is that the ULDM sets the goals and/or decisions and then asks the LLDM for his optima which is calculated separately; the LLDM decisions are then submitted and modified by the ULDM with consideration of the over all benefits of the bi-level problem; and the process continues until a satisfactory solution is reached [9–16].

Most of the developments on BLPP problems focus on bi-level linear programming [14,17,18], and many others focus on bi-level non-linear programming and bi-level multiobjective programming [9–16].

Several BLPP programming approaches and algorithms such as:

1. The hybrid extreme-point search algorithm [19],
2. Mixed-integer problem with complementary slackness [20],
3. The penalty function approach [20,15], and
4. The balance space approach [11–13]

are studied and introduced along with their solution methods.

A bibliography of the related references on bi-level programming in both linear and non-linear cases, which is updated biannually, can be found in [21]. The use of the fuzzy set theory [22] for decision problems with several conflicting objectives was first introduced by Zimmermann [23] Thereafter, various versions of fuzzy programming (FP) have been investigated and widely circulated in literature [20,19,24–27].

Bi-level programming problems have a wide range of applications [28]. Candler and Townsley [29] have suggested applications of bi-level and multi-level programming in governmental problems involving issues such as the setting of penalties for illegal drug import, the fixing of import quotas and the development of transportation and communications infrastructure. Applications to strategic weapons exchange problems and to the distribution of federal budgets among states have been described respectively by Bracken et al. [30] and Cassidy et al. [31]. Anandalingam and Apprey [32] have given a new approach to conflict resolution based on multi-level mathematical programming and have illustrated it with a real world example of the Ganga water conflict problem between India and Bangladesh. In a typical bi-level programming situation, the higher level decision maker is the central government or a central authority which sets policies, and the lower level decision makers are the state governments, industrial managers and the like, who work within the framework of these policies. The bi-level programming structure has been used to model problems concerning spatial competition [33], facility location [34], signal optimization [35] and traffic assignment [36]. Ayed et al. [37] have given a real-world bi-level programming. For example, consider the traffic-planning problem. The planner or the decision-maker, the leader, seeks to improve the performance of the traffic network, while at another level the network users, the followers, make choice with regard to the details of their travel based on the network formed by the planner. Another example is the taxation problem, where the government, the leader, decides the taxation rules and rates, while the taxpayers, the followers, try to minimize their payments based on these rules or rates. Besides the above examples, bi-level programming structure can be found in diverse areas such as economic systems, ecology and environmental studies, biology and chemical engineering, network design, transportation, game theory, data bases, and the theory of classification.

Real-world problems are generally characterized by the presence of many often conflicting and incommensurable objectives and usually are of non-linear nature, which is why we need tools for bi-level non-linear programming capable of handling several conflicting or incommensurable objectives. In this paper, the concept of TOPSIS is further extended to develop a methodology for solving bi-level multi-objective decision-making problems (BL-MODM), and in which the objective functions at each level are non-linear functions to be maximized. The remainder of this paper is organized as follows. Section 2 presents the formulation of bi-level non-linear multi-objective decision-making problems. Section 3 briefly discusses the basic concepts of distance measures of "closeness" and its normalization. The proposed fuzzy TOPSIS approach is developed in Section 4 for solving BL-MODM problems and Section 5 presents the algorithm of the TOPSIS approach for solving BL-MODM problems. The following section presents an illustrative numerical example in order to demonstrate the proposed approach. Finally, the concluding remarks are made in Section 7.

2. Problem formulation

Assume that there are two levels in a hierarchy structure with upper-level decision maker (ULDM) and lower-level decision maker (LLDM). Let the vector of decision variables $x = (x_1, x_2) \in R^n$ be partitioned between the two decision makers. The